Section 4.6

Graphing with Calculus

(1) Curve Sketching

Curve Sketching

The calculus tools we have developed so far enable us to sketch graphs with high accuracy.

Key features of the graph of $f(x)$:

(1) The domain of f — where is f undefined?
(2) Symmetry — is f odd, even, or (usually) neither?
(3) Intervals on which f is increasing or decreasing — use $f'(x)$
(4) Intervals on which f is concave up or down — use $f''(x)$
(5) Local extreme points — use First or Second Derivative Test
(6) Inflection points — points where concavity changes
(7) Horizontal and/or vertical asymptotes

Curve Sketching

Example 1: Sketch the curve of $f(x) = x^4 + 2x^3$.

Example 2: Sketch the curve of $h(x) = \dfrac{2x^2}{x^2 - 1}$.

Example 3: Sketch the curve of $g(x) = x\sqrt{8 - x^2}$.

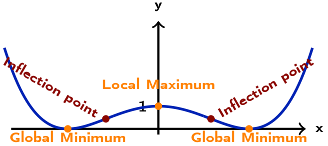
Example 4: Sketch the curve of $r(x) = x - \ln|x|$.

Example 5: Sketch the graph of $k(x) = e^{-x^2}$.

Example 6: Investigate the family of functions $f(x) = cx^4 - 2x^2 + 1$ where c is any real number.

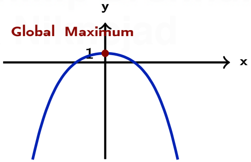
c > 0

- Increasing on $\left(\frac{-1}{\sqrt{c}}, 0\right) \cup \left(\frac{1}{\sqrt{c}}, \infty\right)$.
- Concave up on $\left(-\infty, \frac{-1}{\sqrt{3c}}\right) \cup \left(\frac{1}{\sqrt{3c}}, \infty\right)$.
- Local maximum: $(0,1)$
- Local minima: $x = \pm\frac{1}{\sqrt{c}}$
- Inflection points: $x = \pm\frac{1}{\sqrt{3c}}$



c ≤ 0

- Increasing on $(-\infty, 0)$
- Decreasing on $(0, \infty)$
- Concave down everywhere
- Local max: $(0,1)$
- No inflection points



Example 7: Sketch the curve of f if $f'(x) = (x^2 - 2x)(x-5)^2$.

Domain: $(-\infty, \infty)$ since f is differentiable everywhere.

Asymptotes: None (polynomials don't have asymptotes)

Second derivative:

$$f''(x) = 2(x-5)(2x^2 - 8x + 5)$$

Points of interest: 0, 2, 5 (critical numbers);
$2 + \sqrt{3/2} \approx 3.22$, $2 - \sqrt{3/2} \approx 0.78$, 5 (zeros of f'')

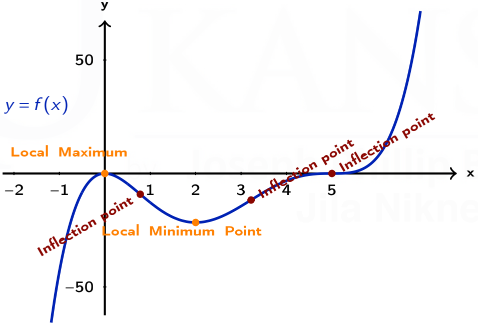
$f''(x)$	−	−	+	+	−	+
$f'(x)$	+	−	−	+	+	+

0	0.78	2	3.22	5
max	i.p.	min	i.p.	i.p.

Example 7: Sketch the curve of f if $f'(x) = (x^2 - 2x)(x-5)^2$.

$f''(x)$	−	−	+	+	−	+
$f'(x)$	+	−	−	+	+	+

0	0.78	2	3.22	5
max	i.p.	min	i.p.	i.p.



Any vertical shift of this graph is also a possibility!